

Problem 1. The loop executes N^2 iterations, and the body of the loop executes in constant time, so the overall running time is $O(N^2)$.

Problem 2. Each loop executes N iterations, and because there are two identical loops, `CODE` executes $2N$ times. Because `CODE` executes in constant time, and because 2 times a constant is still a constant, the overall running time is $O(N)$.

Problem 3. The outer loop executes N iterations. However, the inner loop is dependent on the outer loop, and each time it is reached, executes $0, 1, 2, 3, \dots, N-1$ iterations. The sum

$$\sum_{i=0}^{N-1} i = 0 + 1 + 2 + \dots + (N-1) = \frac{N}{2}(N-1)$$

which (after dropping constant factors and low-order terms) is $O(N^2)$.

Problem 4. The loop executes a constant number of iterations, and the body of the loop executes in constant time. A constant times a constant is a constant, so the overall running time is $O(1)$.

Problem 5. The inner loop is dependent on the outer loop, and executes i^2 iterations each time it is reached, where i is the value of the outer loop's loop variable. So, the total number of times `CODE` is executed is

$$\sum_{i=0}^{(N-1)^2} = 0 + 1 + 4 + 9 + \dots + (N-1)^2 = \frac{(N-1)^3}{2} + \frac{(N-1)^2}{3} + \frac{(N-1)}{6}$$

which is $O(N^3)$. (We will prove this series sum when we cover proof by induction.)

Problem 6. The loop variable i starts at one and doubles on each loop iteration. The final value of i is 2^k , where k is the number of times the body of the loop executes. The loop terminates when $i \geq N$. The smallest value of k such that $2^k \geq N$ is $k = \lceil \log_2 N \rceil$. So, the overall running time is $O(\log_2 N)$, which we can simplify as $O(\log N)$ because all log functions are equivalent in big-O terms, regardless of base.

Problem 7. The innermost loop executes N iterations and the body executes in constant time, so the innermost loop is $O(N)$. The middle loop executes N times, and its body is $O(N)$, so the middle loop is $O(N^2)$. The outer loop executes N times, and its body is $O(N^2)$, so the total running time is $O(N^3)$.